Midterm test for Kwantumfysica 1 - 2011-2012

Friday 30 September 2011, 14:00 - 15:00

READ THIS FIRST:

- Clearly write your name and study number on each answer sheet that you use.
- On the first answer sheet, write clearly the total number of answer sheets that you turn in.
- Note that this test has 5 questions, it continues on the backside of the paper!
- Start each question (number T1, T2,…) on a new side of an answer sheet.
- The test is open book within limits. You are allowed to use the book by Griffiths OR Liboff, and one A4 sheet with notes, but nothing more than this.
- If you get stuck on some part of a problem for a long time, it may be wise to skip it and try the next part of a problem first. The test is only 45 minutes.

Useful formulas and constants:

Electron mass	m_e	$= 9.1 \cdot 10^{-31}$ kg
Electron charge	$-e$	$= -1.6 \cdot 10^{-19}$ C
Planck's constant	h	$= 6.626 \cdot 10^{-34}$ Js $= 4.136 \cdot 10^{-15}$ eVs
Planck's reduced constant	$\hbar$	$= 1.055 \cdot 10^{-34}$ Js $= 6.582 \cdot 10^{-16}$ eVs

Problem T1

For this problem, you must write up your answers in Dirac notation.

Consider a quantum system that contains a charged particle with mass m, that has a time-independent Hamiltonian

$$\hat{H} = \hat{T} + \hat{V} ,$$

where T a kinetic-energy term and V a potential-energy term. The two energy eigenstates of this system with the lowest energy are defined by

$$\begin{aligned} \hat{H}|\varphi_1\rangle &= E_1|\varphi_1\rangle \\ \hat{H}|\varphi_2\rangle &= E_2|\varphi_2\rangle \end{aligned} ,$$

where $E_1 < E_2$ the two energy eigenvalues, and $|\varphi_1\rangle$ and $|\varphi_2\rangle$ two orthogonal, normalized energy eigenvectors. Energy eigenstates with higher energy do not play a role. The observable $\hat{A}$ is associated with the electrical dipole moment A of this quantum system. For this system,

$$\langle\varphi_1|\hat{A}|\varphi_1\rangle = 0 \quad , \quad \langle\varphi_2|\hat{A}|\varphi_2\rangle = 0 \ , \quad \langle\varphi_1|\hat{A}|\varphi_2\rangle = \langle\varphi_2|\hat{A}|\varphi_1\rangle = A_0 \ .$$

Note that the states $|\varphi_1\rangle$ and $|\varphi_2\rangle$ are energy eigenvectors, and that they are ***not*** eigenvectors of $\hat{A}$. At some time defined as $t = 0$, the state of the system is (with all c_n a complex-valued constant)

$$|\Psi_0\rangle = c_1\,|\varphi_1\rangle + c_2\,|\varphi_2\rangle = \frac{1}{\sqrt{2}}|\varphi_1\rangle - \frac{1}{\sqrt{2}}|\varphi_2\rangle \ .$$

Show that as a function of time $t > 0$, the expectation value for $\langle\hat{A}\rangle$ has oscillations at one frequency only. Determine this frequency and the oscillation amplitude (expressed in the constants that are mentioned above), for the case that the system is in $|\Psi_0\rangle$ at $t = 0$.

Problem T2

An electron beam hits a wall with two parallel slits 1 and 2. A detector is located in some point P behind the wall with the two slits. The wavefunction Ψ_1 represents the electron state for electrons that arrive at the detection point P, for the case that the electron went through slit 1 and hits the detector. Similarly, Ψ_2 represents the state at P for the case that an electron went through slit 2. When only slit 1 is open, the detector P counts on average $N_1 = 100$ electrons per second. Due to an asymmetry in the system $|\Psi_2/\Psi_1| = a = 3.0$. What is the count rate of electrons in the detector at P when:

a) only slit 2 is open.

b) both slits are open and there is an interference maximum at point P.

c) both slits are open and there is an interference minimum at point P.

Problem T3

A particle of the mass m is moving in a one-dimensional potential $U(x) = kx^2/2$ (with $k > 0$, note that this system is a simple harmonic oscillator). Assume that the wave function of the ground state of this system is of the form of $\Psi(x) = A \exp(-\alpha x^2)$, where A and α are some constants. Use the time-independent Schrödinger equation to find the constant α and show that the energy E of this state is equal to

$$E = \tfrac{1}{2}\hbar\omega_0 \quad \text{with} \quad \omega_0 = \sqrt{\frac{k}{m}}$$

(ω_0 is here the classical angular frequency of oscillations).

Hint: fill $\Psi(x)$ in into the time-independent Schrödinger equation, simplify the equation, and note that the equation must hold for all x.

Problem T4

Assume that an operator $\hat{A}$ has the eigenfunctions φ_n and associated eigenvalues a_n. Prove that for any function f the next statement holds: $f(\hat{A})\ \varphi_n = f(a_n)\ \varphi_n$

Hint: use that f can be written as a power series

$$f(\hat{A}) = \sum_{l=0}^{\infty} c_l (\hat{A}^l)$$

and build your proof by checking the statement for each term.

Problem T5

A wide parallel beam of electrons (only motion in y-direction) with a velocity of $v_y = 600$ m/s is incident on a screen with a single narrow slit of with d. Behind this first screen there is a second screen for detection. The distance between the screen with the slit and the detection screen is $l = 1$ m. Using the uncertainty principle, make an *estimate* for the width of the slit d for which the width W of the image on the detection screen is the narrowest.

Hint: For electrons that just passed the screen, you can assume that for transverse motion in the beam the state of electrons is close to a state with minimum uncertainty.